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Solvability, Consistency and the Renormalization Group in Large-N_c Models of Hadrons

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Abstract

We establish the following fundamentals about Lagrangian models of meson-baryon interactions in the large-N_c limit: **1.** Calculating the leading-order contribution to 1-meson/2-baryon Green's functions in the $1/N_c$ expansion involves summing an infinite class of divergent Feynman diagrams. So long as the bare Lagrangian properly obeys all large-N_c selection rules, this all-loops resummation is accomplished exactly by solving coupled classical field equations with a short-distance cutoff. **2.** The only effect of the resummation is to renormalize the bare Yukawa couplings, baryon masses and hyperfine baryon mass splittings of the model. **3.** In the process, the large-N_c renormalization group flow of these bare parameters is completely determined. We conjecture that variants of the Skyrme model emerge as UV fixed points of such flows.

Introduction. In the two decades since 't Hooft's original suggestion [1], little progress has been made towards a solution of QCD in the limit $N_c \to \infty$. However, several powerful constraints on the allowed spectrum of baryons and their interactions with mesons in this limit are known. These "large-N_c selection rules" can be derived either from the planar graphs in the underlying quark-gluon theory [2, 3, 4], from the Skyrme model [5, 6], from the nonrelativistic quark model [7, 8], or finally from the self-consistency of meson-baryon Feynman diagrams [9, 10, 11, 12].

In this Letter, we show how all these constraints can be incorporated consistently into an effective hadron Lagrangian. Although meson self-interactions become weak as $N_c \to \infty$, meson-baryon Yukawa couplings become strong, and an infinite number of diagrams dress the bare Yukawa vertices of the theory at *leading* order in $1/N_c$. Our first result, extending Ref. [13], is that these diagrams can be summed exactly by solving coupled classical field equations in the presence of a nonrelativistic baryonic source; i.e., these models are *solvable* in the $N_c \to \infty$ limit. Moreover, the only effect of this resummation is to renormalize the bare baryon masses, hyperfine mass splittings, and on-shell Yukawa couplings of the theory, in such a way that the large-N_c constraints incorporated in the bare Lagrangian emerge unscathed at the renormalized level. This confirms the *consistency* of effective hadron Lagrangians at large-N_c. Finally, the all-loops *Renormalization Group (RG) equations* controlling the running of these bare quantities with the UV cutoff Λ falls out as a by-product of the above analysis. The fact that classical equations always suffice to leading order is a reflection of the important fact that N_c appears in the action in the combination $\hbar/N_c$.

The resulting picture of the large-N_c baryon[1] is reminiscent of the old chiral bag models, in which explicit nucleon/quark degrees of freedom for $r \leq \Lambda^{-1}$ are matched onto a cloud of hedgehog pions at $r \geq \Lambda^{-1}$ [14]. But whereas historically these bag models were motivated by chiral symmetry, our current construction follows from large-N_c *alone*—with no reference whatsoever to approximate chiral invariance. And since at large distances our meson-dressed baryon is indistinguishable from a skyrmion, it is natural to conjecture variations of the Skyrme model as arising as UV fixed points of the RG flow as $\Lambda \to \infty$.

Large-N_c hadron models. We study generic 2-flavor relativistic hadron Lagrangians that conserve C, P, T, and isospin, and are further restricted *only* by these five large-N_c consistency conditions:

1: Straightforward quark-gluon counting arguments show that n-meson vertices $\sim N_c^{1-\frac{n}{2}}$, as do n-meson 2-baryon vertices [2, 3, 4]. Thus, baryon masses ($n = 0$) and Yukawa couplings ($n = 1$) grow like N_c and $\sqrt{N_c}$, respectively.

[1] This picture, in which a hard core of explicit nucleon degrees of freedom is dressed by tree graphs (Born graphs) of mesons, is sometimes referred to as "hard-core Born-ography."

2: The 2-flavor baryon spectrum of large-N_c QCD consists of an infinite tower of positive parity states with $I = J = 1/2,\ 3/2,\ 5/2\ldots$. To leading order these states are degenerate, with mass $M_{\rm bare} \sim N_c$ [3, 4, 5, 9, 11].

3: Hyperfine baryon mass splittings have the form $J(J+1)/2\mathcal{I}_{\rm bare}$ where $\mathcal{I}_{\rm bare} \sim N_c$ [3, 4, 5, 12].

4: Yukawa couplings are constrained to obey the "proportionality rule" [5, 6, 8, 9, 11], which fixes the interaction strength of a given meson with each member of the baryon tower as a multiple of one overall coupling constant (e.g., $g_{\pi N\Delta}/g_{\pi NN} = 3/2$).

5: Finally, the allowed couplings of mesons to the baryon tower must obey the $I_t = J_t$ rule [4, 6, 8, 10]; e.g., the ρ meson must be tensor-coupled to the nucleon while the ω meson is vector-coupled at leading order in $1/N_c$, in good agreement with phenomenology.

Summing the leading-order graphs. The N_c dependence of coupling constants described in **1** above suffices to identify the leading-order meson-baryon Feynman graphs for any given physical process. Thus, purely mesonic processes are dominated by meson tree graphs, which vanish as $N_c \to \infty$. Correspondingly, meson-baryon processes are dominated by those graphs which become meson trees if the baryon lines are removed. To illustrate the complexity of such graphs, look at a typical multi-loop correction, Fig. 1b, to the bare Yukawa coupling, Fig. 1a. Since, by design, the graph in Fig. 1b contains no loops formed purely from mesonic legs, this graph scales like $\sqrt{N_c}$ just like the bare vertex. This is trivially checked by multiplying together all the vertex constants and ignoring propagators entirely, since both meson and baryon propagators $\sim N_c^0$. Hence, in order to calculate the dressed Yukawa vertex to leading order, one must sum this infinite set of diagrams. One must *also* sum all multiple insertions of the baryon self-energy corrections and additional vertex corrections as illustrated in Fig. 1d, as these too contribute at leading order [3, 12]. Since many of the loop integrations in these diagrams are UV divergent, it is necessary to regulate the theory with a UV cutoff Λ.

Following the methods of Ref. [13], a summation of *all* such graphs is possible, to leading order in $1/N_c$. To see how, we focus initially on a model of baryons and pions only, in which the above five conditions are imposed:

$$\begin{aligned}\mathcal{L} \;&=\; \frac{1}{2}(\partial_\mu\vec{\pi})^2 - \frac{m_\pi^2}{2}\vec{\pi}^2 - V(\vec{\pi}) + \bar{N}(i\partial\!\!\!/ - M_N)N \\ &\qquad - g_\pi^{\rm bare}\partial_\mu\pi^a\bar{N}\gamma^5\gamma^\mu\tau^a N + (\text{higher-spin baryons}) \qquad (1)\end{aligned}$$

The incorporation of additional meson species will be discussed below. Here V is a general pion potential including quartic and higher vertices, and $M_N = M_{\rm bare} + 3/8\mathcal{I}_{\rm bare}$ including the hyperfine splitting. The pseudovector form of the πN coupling is determined by the $I_t = J_t$ rule while the proportionality rule fixes the corresponding pion couplings to the higher-spin baryons.

From the position-space Feynman rules for (1), the sum of all the graphs such as Fig. 1b is formally given by:

$$\sum_{n=1}^{\infty}\int\left(\prod_{i=1}^{n}d^4w_i d^4z_i\right) Blob_n(x;w_1,\ldots,w_n)\sum_{\rho\in S_n}\prod_{i=1}^{n}g(w_i,z_{\rho(i)})\cdot\mathcal{Y}(z_{\rho(i)})\prod_{i=1}^{n-1}G(z_i,z_{i+1}) \tag{2}$$

$Blob_n$ denotes the shaded blob in Fig. 1c; the sum over permutations $\rho\in S_n$ counts the $n!$ possible tanglings of the n meson lines which connect to the blob. All isospin and spin indices have been suppressed in (2); g_{ab} and G_J are the position-space meson and spin-J baryon propagators, respectively, and $\mathcal{Y}^a_{JJ'}$ is the appropriate pseudovector Yukawa vertex factor.

Passing to the large-N_c limit, we can exploit two important simplifications to this expression. First, the baryons become very massive and can be treated nonrelativistically. For forward time-ordering, $z_0 < z_0'$, the baryon propagator $G(z,z')$ can be replaced by its nonrelativistic counterpart $G_{NR}(z,z')+\mathcal{O}(1/N_c)$. As usual, the reversed or "Z-graph" time ordering $z_0' < z_0$ contributes effective pointlike vertices in which two or more mesons couple to the baryon at a single point. These effective "seagull" terms are naturally grouped with similar pointlike interactions that one may wish to incorporate from the outset in the bare Lagrangian, and we will mention below the simple generalization of our analysis required to account for both these sets of terms consistently.

A second simplification comes from switching to the $SU(2)$ "collective coordinate basis" $|A\rangle$ for the $I=J$ baryon tower [5, 7, 8], related to the more familiar spin-isospin basis $|I=J,i_z,s_z\rangle$ via

$$\langle I=J,i_z,s_z|A\rangle=(2J+1)^{1/2}D^{(J)}_{-s_z,i_z}(A^\dagger)\cdot(-)^{J-s_z} \tag{3}$$

with $D^{(J)}(A)$ a Wigner D-matrix. In the $|A\rangle$ basis, the baryon propagator can be expressed as a *quantum mechanical* path integral over two collective coordinates: $\mathbf{X}$, representing the position of the center of the baryon, and A, describing its $SU(2)$ (iso)orientation,

$$\begin{aligned} G^{A,A'}_{NR}(z,z') &= \theta(z_0'-z_0)\int_{\mathbf{X}(z_0)=\mathbf{z}}^{\mathbf{X}(z_0')=\mathbf{z}'}\mathcal{D}\mathbf{X}(t)\int_{A(z_0)=A}^{A(z_0')=A'}\mathcal{D}A(t) \\ &\quad\times\exp\left(i\int_{z_0}^{z_0'}dt\,M_{\rm bare}+\frac{1}{2}M_{\rm bare}\dot{\mathbf{X}}^2+\mathcal{I}_{\rm bare}{\rm Tr}\dot{A}^\dagger\dot{A}\right) \end{aligned} \tag{4}$$

The path integration over $A(t)$ can be performed using the beautiful result of Schulman for free motion on the $SU(2)$ group manifold [15],

$$\begin{aligned} &\int_{A(z_0)=A}^{A(z_0')=A'}\mathcal{D}A(t)\,\exp i\int_{z_0}^{z_0'}dt\,\mathcal{I}_{\rm bare}{\rm Tr}\dot{A}^\dagger\dot{A} \quad = \\ &\sum_{J=1/2,\,3/2,\cdots}\ \sum_{i_z,s_z=-J}^{J}\langle A'|J,i_z,s_z\rangle\cdot\exp i(z_0'-z_0)\frac{J(J+1)}{2\mathcal{I}_{\rm bare}}\cdot\langle J,i_z,s_z|A\rangle\ , \end{aligned} \tag{5}$$

yielding the conventional nonrelativistic propagator for an infinite tower of particles with masses $M_{\rm bare}(J)=M_{\rm bare}+J(J+1)/2\mathcal{I}_{\rm bare}$ as required.

The greatest advantage of the $|A\rangle$ basis is that the Yukawa vertex factor $\mathcal{Y}$ for the pion-baryon coupling becomes *diagonal* [5, 7, 8]:

$$\mathcal{Y}^a_{A,A'}(z) = -3g_\pi^{\rm bare} D^{(J)}_{ab}(A)\delta(A-A')\frac{\partial}{\partial z_b} \tag{6}$$

Substituting for G and $\mathcal{Y}$ in (2), we find that this diagonality property allows us trivially to perform the sum over the $n!$ products of step functions, which collapses to unity. Interchanging the order of path integration and the product over baryon legs, one obtains

$$\int \mathcal{D}\mathbf{X}(t)\mathcal{D}A(t)\sum_{n=1}^{\infty}\int\left(\prod_{i=1}^{n} d^4w_i d^4z_i\right) Blob_n(x;w_1,\ldots,w_n)$$
$$\times \prod_{i=1}^{n} g(w_i,z_i)\cdot\mathcal{Y}(z_i)\delta^{(3)}_\Lambda(\mathbf{z}_i-\mathbf{X}(z_i^0))\exp iS_{\rm baryon}[\mathbf{X},A] \tag{7}$$

where $S_{\rm baryon}$ is short for the exponent of (4). We have reduced the problem to a sum of *tree* diagrams for the pions interacting with the baryon collective coordinates through a δ-function source. The only remaining manifestation of the UV cutoff Λ is that this δ-function should be smeared out over a radius $\sim\Lambda^{-1}$, as denoted by δ_Λ in (7), which we assume still preserves rotational invariance.

In short, the massive baryon has become a translating, (iso)rotating, smeared point-source for the pion field, the effect of which can be found be solving the appropriate *classical* Euler-Lagrange equation for a configuration we call $\vec{\pi}_{\rm cl}(x;[\mathbf{X}],[A])$ [13]:

$$(\Box + m_\pi^2)\pi^a_{\rm cl} + \frac{\partial V}{\partial \pi^a_{\rm cl}} = 3g_\pi^{\rm bare}D^{(1)}_{ai}(A(t))\frac{\partial}{\partial x^i}\delta^{(3)}_\Lambda(\mathbf{x}-\mathbf{X}(t)) \tag{8}$$

It is easily checked (Fig. 2) that the order-by-order perturbative solution of Eq. (8) generates precisely the sum of graphs appearing in (7). By similar semi-classical reasoning (Fig. 3), the leading-order parts of the additional vertex and self-energy corrections highlighted in Fig. 1d exponentiate exactly, and are correctly accounted for by evaluating the mesonic plus Yukawa pieces of the action (call this sum $S_{\rm eff}$) on $\vec{\pi}_{\rm cl}$. The final leading-order result for the complete sum of graphs contributing to the dressed pion-baryon vertex is

$$\int \mathcal{D}\mathbf{X}(t)\mathcal{D}A(t)\,\pi^a_{\rm cl}(x;[\mathbf{X}],[A])\exp i(S_{\rm baryon}+S_{\rm eff}[\vec{\pi}_{\rm cl},\mathbf{X}(t),A(t)]\,) \tag{9}$$

Solving the classical field equation. We solve Eq. (8) by relating it to the analogous equation for the *static* pion cloud, $\vec{\pi}_{\rm stat}(\mathbf{x})$, surrounding a fixed baryon source ($\mathbf{X}(t)\equiv\mathbf{0}, A(t)\equiv 1$):

$$(-\nabla^2 + m_\pi^2)\pi^a_{\rm stat} + \frac{\partial V}{\partial \pi^a_{\rm stat}} = 3g_\pi^{\rm bare}\frac{\partial}{\partial x^a}\delta^{(3)}_\Lambda(\mathbf{x}) \tag{10}$$

The solution will generically have the hedgehog form familiar from the Skyrme model: $\pi^a_{\rm stat}(\mathbf{x}) = (f_\pi x^a/2r)F(r)$ where $r=|\mathbf{x}|$. The profile function $F(r)$ is found, in turn, by solving the induced

nonlinear radial ODE. While its detailed form depends sensitively on the potential $V(\vec{\pi})$, its asymptotic behavior for large r is fixed by the linearized field equation,

$$F(r) \quad \longrightarrow \quad \mathcal{A}\left(\frac{m_\pi}{r} + \frac{1}{r^2}\right) e^{-m_\pi r} \tag{11}$$

where the constant $\mathcal{A}$ must be extracted numerically. The solution to (8) is then simply given, up to $1/N_c$ corrections, by translating and (iso)rotating $\vec{\pi}_{\rm stat}$:

$$\pi^a_{\rm cl}(x; [\mathbf{X}], [A]) = D^{(1)}_{ab}(A(t))\pi^b_{\rm stat}(\mathbf{x} - \mathbf{X}(t)) \tag{12}$$

The additional collective coordinate dependence carried by $\vec{\pi}_{\rm cl}$ versus $\vec{\pi}_{\rm stat}$ is precisely that required for overall isospin, angular momentum and 4-momentum conservation, as is easily checked [16].

We seek the renormalized on-shell πN interaction, to leading order in $1/N_c$. It is defined in the usual way as the on-shell residue of the LSZ amputation of the full set of graphs that are summed implicitly by Eq. (9). Formally, this amputation is identical to the procedure one follows in the Skyrme model [16]. In particular, the physically correct analytic structure of the one-point function follows from the $1/N_c$ corrections to $\vec{\pi}_{\rm cl}$ which describe its response to the rotation of the source. (The specifics of this response, involving an interesting small distortion *away* from the hedgehog ansatz [16], need not concern us here.) Thanks to the (iso)vector nature of the hedgehog, the resulting S-matrix element for one-pion emission defines a renormalized on-shell pseudovector interaction of the pion with the baryon tower, *identical* to the bare interaction in (1), except for the coupling constant renormalization $g_\pi^{\rm bare} \to g_\pi^{\rm ren}$. Again as in the Skyrme model, this latter quantity is determined by the asymptotics of $\vec{\pi}_{\rm stat}$, Eq. (11), and is explicitly given by [5, 16] $g_\pi^{\rm ren} = (2/3)\pi f_\pi \mathcal{A}$. Thus the proportionality and $I_t = J_t$ rules for the pion-baryon coupling remain true at the renormalized level, as claimed.

Furthermore, the result of evaluating $S_{\rm eff}[\vec{\pi}_{\rm cl}]$ is just an additive renormalization of the bare parameters of $S_{\rm baryon}$, due to the meson cloud:

$$S_{\rm baryon} + S_{\rm eff}[\vec{\pi}_{\rm cl}, \mathbf{X}, A] \;=\; \int dt \left(M_{\rm ren} + \frac{1}{2} M_{\rm ren}\dot{\mathbf{X}}^2 + \mathcal{I}_{\rm ren}{\rm Tr}\dot{A}^\dagger \dot{A} \right) \tag{13}$$

where

$$M_{\rm ren} = M_{\rm bare} + \int d^3\mathbf{x}\,(\nabla\pi^a_{\rm cl})^2 \ , \qquad \mathcal{I}_{\rm ren} = \mathcal{I}_{\rm bare} + \frac{2}{3}\int d^3\mathbf{x}\,\vec{\pi}^2_{\rm cl} \tag{14}$$

It follows that $M_{\rm ren}(J) = M_{\rm ren} + J(J+1)/2\mathcal{I}_{\rm ren}$ and so the form of the hyperfine mass splitting is likewise preserved by renormalization.

The generalization of the above analysis to models including several species of mesons involves solving the coupled classical radial ODE's for all the meson fields, using generalized hedgehog ansatze familiar from vector-meson-augmented Skyrme models. A particularly rich meson model

might include, in addition to the pion, the tensor-coupled ρ, i.e., $g_\rho^{\rm bare}\partial_\mu\vec{\rho}_\nu\cdot\bar{N}\sigma^{\mu\nu}\vec{\tau}N$, the vector-coupled ω, i.e., $g_\omega^{\rm bare}\omega_\mu\bar{N}\gamma^\mu N$, and/or the "$\sigma$-meson," which couples simply as $g_\sigma^{\rm bare}\sigma\bar{N}N$. Again, on shell, the form of these particular couplings survives renormalization. Multi-meson "sea-gulls," defined earlier, can also be incorporated. For Λ sufficiently large, these have the effect of multiplying the right-hand side of Eq. (8) by a function of the field, which simply leads to further *algebraic* renormalizations of the bare Yukawa constants.

Large-N_c Renormalization Group. We have described an explicit numerical procedure for calculating the renormalized Yukawa couplings, baryon masses and hyperfine mass splittings, to leading order in $1/N_c$, directly from the classical meson cloud surrounding the baryon. Since the δ-function source on the right-hand side of Eq. (8) is smeared out over a characteristic length Λ^{-1}, these quantities depend explicitly on Λ. In order to hold the physical, renormalized masses and couplings fixed, it is necessary to vary simultaneously *both* Λ and the corresponding bare quantities. This procedure defines an RG flow for $M_{\rm bare}(\Lambda)$, $\mathcal{I}_{\rm bare}(\Lambda)$ and $g^{\rm bare}_{\pi,\rho,\omega,\sigma}(\Lambda)$, valid to all orders in the loop expansion but strictly to leading order in $1/N_c$. Numerical work along these lines is in progress.

It is particularly interesting to speculate whether these RG flows exhibit a UV fixed point, meaning that the theory has a nontrivial continuum limit. Such a limit—*if* it exists—must be defined with care. As the validity of nonrelativistic baryon propagators breaks down when the momenta of the impinging mesons $\sim M_N$, it is natural to restrict $\Lambda \ll M_N \sim N_c$; e.g., $\Lambda \sim \sqrt{N_c}$. Basically, this is tantamount to only taking the continuum limit $\Lambda \to \infty$ *after* letting $N_c \to \infty$, an important caveat, as it is likely that these limits do not commute. Moreover, if $\Lambda \to \infty$ while the bare Yukawa couplings stay finite, the nonlinearity of Eq. (8) ensures that it no longer has a meaningful solution. This suggests that the bare Yukawa couplings must flow to zero in this limit. Consequently, the continuum theory is nontrivial if the *homogeneous* variant of Eq. (10) (with the right-hand side set to zero) admits a nonperturbative solution, i.e., a soliton/skyrmion (either energetic or topological). We are left with the conjectural but conceptually pleasing picture of variants of the Skyrme model emerging, in large-N_c, as UV fixed points of a more pedestrian class of models that have dressed explicit fields representing the nucleons, Δ's, etc. Note that purely mesonic couplings and masses do *not* flow in our program, since meson loops $\sim 1/N_c$. This suggests that we use the experimental, renormalized meson parameters from the outset—implicitly summing all such loops! Note that only for a meson parameter space of measure zero can there be a UV fixed point of the type just described, since, obviously, in Skyrme-type models the renormalized meson parameters, Yukawa couplings, and baryon masses cannot be independently fixed, but are tightly interconnected (which is precisely the point of the Skyrme approach).

Meson-baryon scattering. The above formalism for dressed Yukawa couplings is naturally extended to physical processes in which the baryon interacts with more than one asymptotic meson. Figures 4a and 4b illustrate all the leading-order contributions, "Compton-type" versus "exchange-type" respectively, to meson-baryon scattering. After resummation/renormalization, the Compton-type graphs turn into the two elementary time-ordered graphs 5a and 5b, where

now the vertices are the *renormalized* Yukawa couplings. The exchange-type graphs sum to Fig. 5c, where the thick meson line stands for the propagation of the fluctuating meson field, say $\delta\vec{\pi}$ (defined as $\vec{\pi} - \vec{\pi}_{\rm cl}$), through the nontrivial background generated by $\vec{\pi}_{\rm cl}$ itself [13]. It is pleasing that, despite appearances, Figs. 5a-c can be viewed in a unified semiclassical manner, as we explained in Ref. [16] (Sec. 7).

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References

[1] G. 't Hooft, *Nucl. Phys.* **B72** (1974) 461 and **B75** (1974) 461.

[2] E. Witten, *Nucl. Phys.* **B160** (1979) 57.

[3] M. A. Luty and J. March-Russell, hep-ph/9310369; M. A. Luty, hep-ph/9405271.

[4] C. Carone, H. Georgi, L. Kaplan and D. Morin, hep-ph/9406277; C. Carone, H. Georgi and S. Osofsky, *Phys. Lett.* **B322** (1994) 227.

[5] G. Adkins, C. Nappi and E. Witten, *Nucl. Phys.* **B228** (1983) 552.

[6] M. P. Mattis and M. Mukerjee, *Phys. Rev. Lett.* **61** (1988) 1344.

[7] A. V. Manohar, *Nucl. Phys.* **B248** (1984) 19.

[8] M. P. Mattis and E. Braaten, *Phys. Rev.* **D39** (1989) 2737.

[9] J. Gervais and B. Sakita, *Phys. Rev.* **D30** (1984) 1795.

[10] M. P. Mattis, *Phys. Rev.* **D39** (1989) 994 and *Phys. Rev. Lett.* **63** (1989) 1455.

[11] R. Dashen and A. V. Manohar, *Phys. Lett.* **B315** (1993) 425 and 438; R. Dashen, E. Jenkins and A. V. Manohar, *Phys. Rev.* **D49** (1994) 4713; E. Jenkins and A. V. Manohar, hep-ph/9405431.

[12] E. Jenkins, *Phys. Lett.* **B315** (1993) 431.

[13] P. Arnold and M. P. Mattis *Phys. Rev. Lett.* **65** (1989) 8311; M. P. Mattis and R. Silbar, hep-ph/9405366.

[14] A. Chodos and C. Thorn, *Phys. Rev.* **D12** (1975) 2733; M. Rho, A. S. Goldhaber and G. E. Brown, *Phys. Rev. Lett.* **51** (1983) 747.

[15] L. S. Schulman *Phys. Rev.* **176** (1968) 1558.

[16] N. Dorey, J. Hughes and M. P. Mattis, hep-ph/9404274, and *Phys. Rev.* **D49** (1994) 3598.

Figure Captions

1. (a) The bare meson-baryon coupling, which we shall refer to generically as a "Yukawa coupling." Henceforth, directed lines are baryons, undirected lines are mesons. Internal baryon lines must be summed over all allowed states in the $I = J$ tower. (b) A typical multi-loop dressing of (a) that contributes at *leading* order, $N_c^{1/2}$, as it contains no purely mesonic loops. (c) A systematic counting of the diagrams such as (b). The shaded blob contains only tree-level meson branchings. There are $n!$ distinct "tanglings" of the attachments of the shaded blob to the baryon line. (d) A typical dressing such as (b), augmented by additional baryon self-energy and vertex corrections, all of which also contribute at leading order.

2. The graphical perturbative solution to Eq. (8) as a sum of tree-level one-point functions terminating in the effective Yukawa vertex.

3. Diagrammatic representation of $S_{\rm eff}(\vec{\pi}_{\rm cl})$. When combined with the expansion depicted in Fig. 2, $\exp iS_{\rm eff}$ combinatorically correctly accounts for all the leading-order baryon self-energy and vertex corrections highlighted in Fig. 1d.

4. Typical "Compton-type" (a) and "exchange-type" (b) leading-order contributions to meson-baryon scattering. In an exchange-type graph, one can trace a path from the incoming meson ϕ to the outgoing meson ϕ' without ever traversing a baryon line segment; in a Compton-type graph one cannot.

5. The three leading-order *renormalized* contributions to meson-baryon scattering, which account for all the graphs of the type illustrated in Fig. 4.

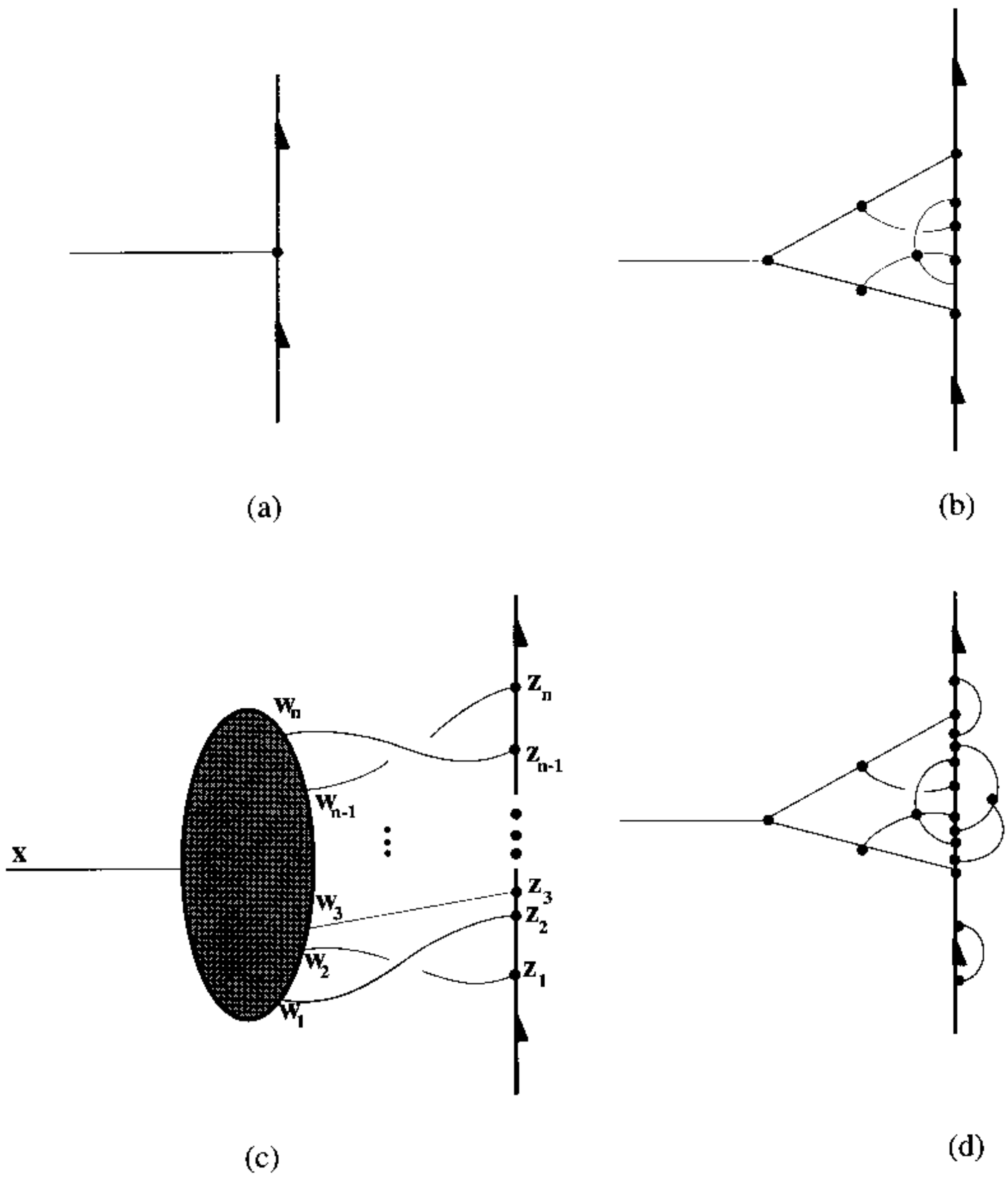
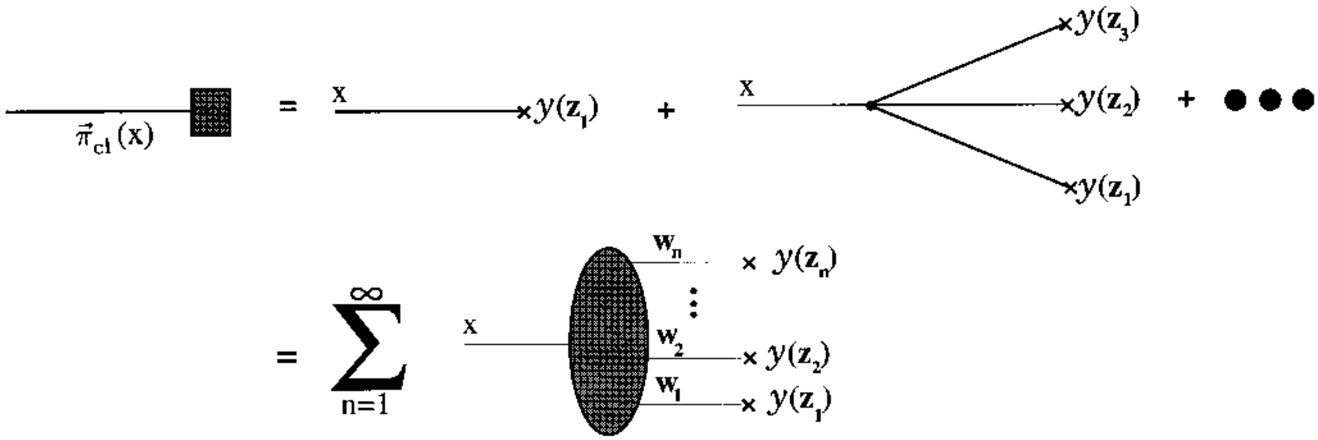
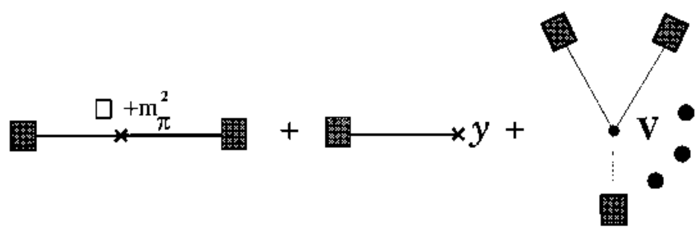


Fig. 1



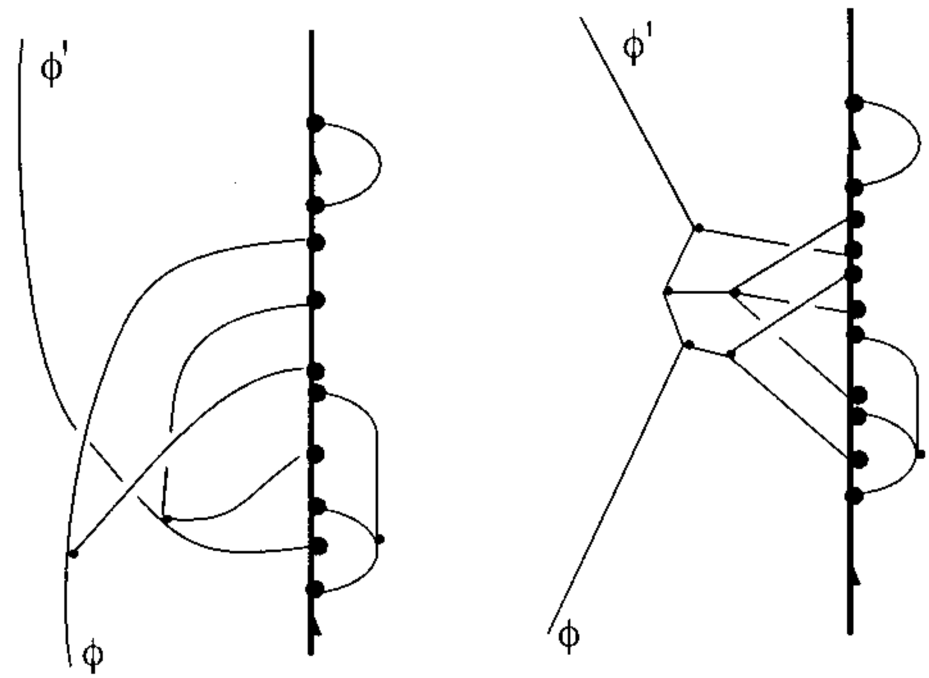
$\vec{\pi}_{cl}(x)$
=
x
$y(z_1)$
+
x
$y(z_3)$
$y(z_2)$
$y(z_1)$
+
=
$\sum_{n=1}^{\infty}$
x
w_n
$y(z_n)$
w_2
$y(z_2)$
w_1
$y(z_1)$

Fig. 2



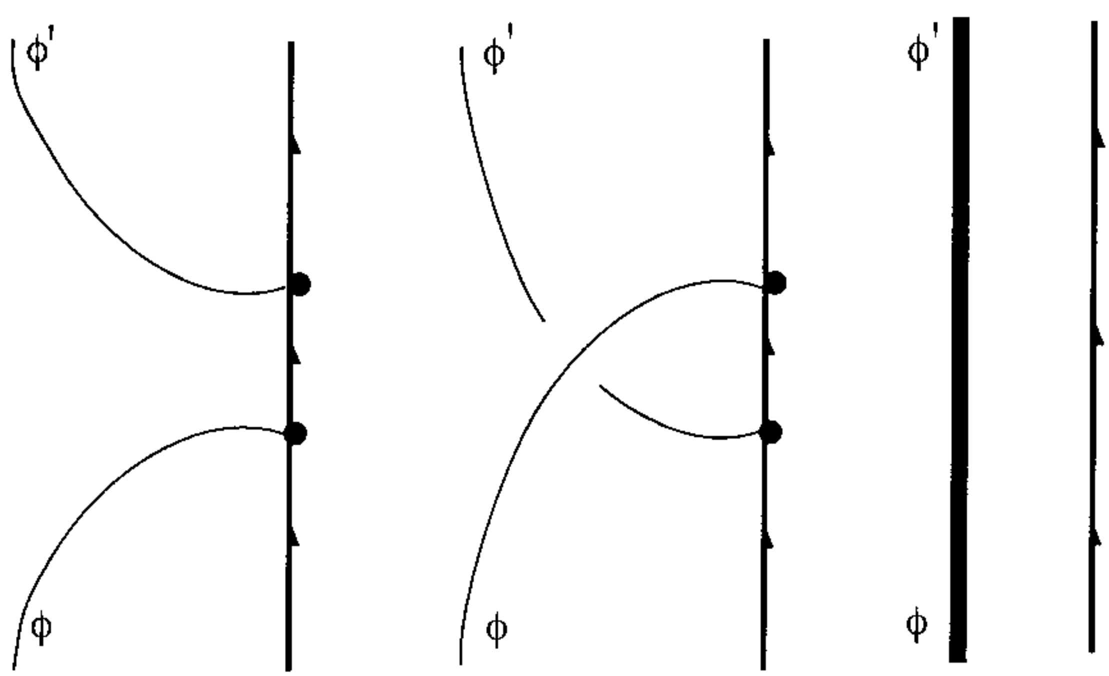
$\Box + m_\pi^2$
+
y
+
V

Fig. 3



(a) (b)

Fig. 4



(a) (b) (c)

Fig. 5